

Optimal income taxation of Polish low earners

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Following Mirrlees (1971), Diamond (1998), Saez (2001)

$$\max_{\tau(\cdot)} W = \int_{a_L}^{a_H} G(U_a) f(a) da$$

subject to

$$U_a(y_a) = y_a - \tau(y_a) - \frac{1}{1+\eta} \left(\frac{y_a}{a}\right)^{1+\eta}$$

$$y_a^\eta = a^{1+\eta} [1 - \tau'(y_a)]$$

$$R = \int_{a_L}^{a_H} \tau(y_a) f(a) da$$

where:

$a \in [a_L, a_H]$ – agent's productivity;

$f(a)$ – PDF (the distribution of agents' skills, which is unobservable);

G – increasing and concave function of preferences of the policy maker with regard to redistribution (social welfare);

U_a – agent's realized utility;

$y_a = aL$ – agent's labor income (so $L = \frac{y_a}{a}$);

τ – tax function;

η – inverse of the elasticity of labor supply with respect to the wage;

Here we identify $G(U) = \frac{U^{1-\theta}-1}{1-\theta}$, where $\theta \in [0,1]$ captures the preferences of the policy maker with respect to redistribution. Naturally, as θ increases the motive for redistribution becomes stronger.

Hence, our optimal taxation problem involves two parameters η and θ . Furthermore, the solution is contingent on the distribution of skills described with $f(a)$. However, we know the actual distribution of income in Poland (Fig. 1).

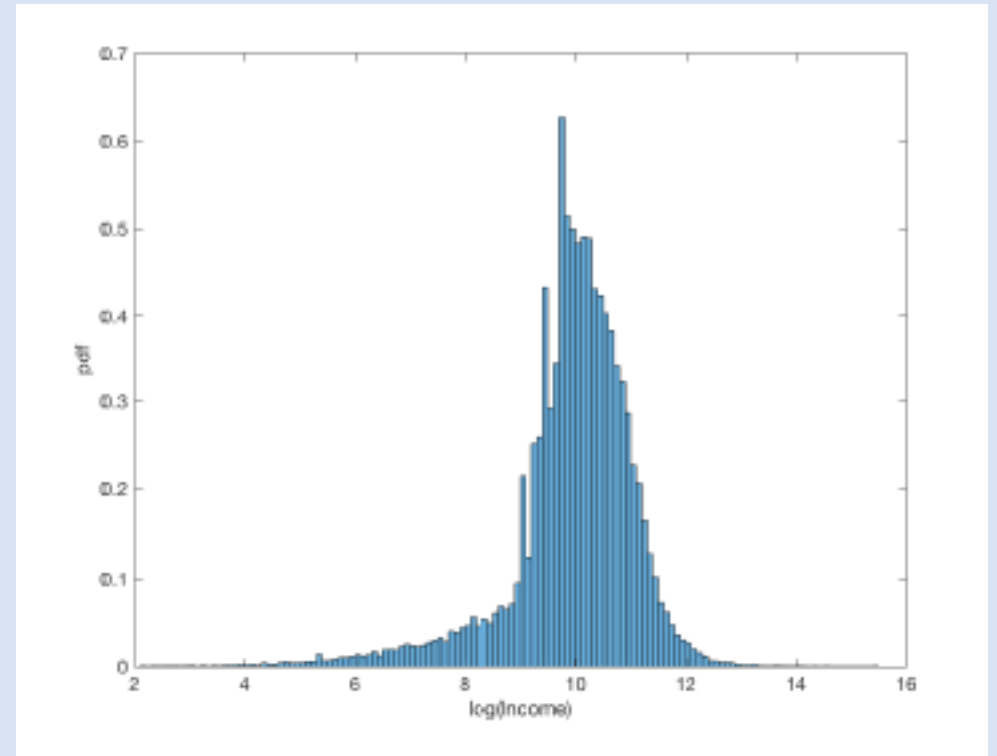


Figure 1. Sample of 50,000 personal income taxpayers (MF, 2016)

We consider a simple piece-wise linear form of $\tau(\cdot)$

$$\tau(y) = \begin{cases} T + \tau_1 y & \text{for } y \leq \bar{y} \\ T + \tau_1 \bar{y} + \tau_2 (y - \bar{y}) & \text{for } y > \bar{y} \end{cases}$$

where T , τ_1 , and τ_2 are constants.

Real life data together with the actual form of taxation in Poland

$$\tau(y) = \begin{cases} 0.18y & \text{for } y \leq 85528 \\ 0.18 * 85528 + 0.32 * (y - 85528) & \text{for } y > 85528 \end{cases}$$

allow us for the determination of the actual distribution of skills $f(a)$ for a given value of η , given the actual distribution of income depicted in Fig. 1.

Now we are to choose four numbers T , τ_1 , τ_2 , and \bar{y} to maximize the welfare function W subject to the government budget constraint where the revenue requirement, R , is equal to the value empirically observed in the actual sample of the Polish tax returns.

We can make some general observations. As η increases, i.e. the elasticity of labor supply falls, the magnitude of basic income, T , increases. Furthermore, the marginal tax rates for $\theta = 0.99$ fall when the elasticity of labor supply increases. Finally, the optimal tax function – in this family of functions – is in fact regressive.

In general, the shape of the optimal tax function is very sensitive to the values of the underlying parameters.

Please note that the results (Tab. 1&2) have been obtained after all observations with income level equal to 0 were removed from the data set.

η	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$
T	-8729	-5823	-3996	-4693	-4209
τ_1	0.72	0.69	0.70	0.99	0.71
τ_2	0.44	0.32	0.21	0.14	0.09
\bar{y}	7200	7200	7800	7200	9000

Table 1: Optimal Tax Function for $\theta = \frac{1}{2}$

η	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$
T	-10600	-7510	-5301	-3976	-3560
τ_1	0.74	0.71	0.7	0.62	0.41
τ_2	0.53	0.4	0.27	0.18	0.11
\bar{y}	9200	9000	9400	10400	14600

Table 2: Optimal Tax Function for $\theta = 0.99$

If one chooses not to ignore observations with 0 income in the sample, the results substantially change (Tab. 3).

η	4	1	$\frac{1}{2}$
T	-4938	-512	-13
τ_1	0.15	0.1	0.15
τ_2	0.5	0.27	0.25
\bar{y}	30000	29000	92000

Table 3. Optimal tax function for $\theta = \frac{1}{2}$